On - line Scheduling of Hybrid Chemical Plants with Parallel Production Lines and Shared Resources: a Feedback Control Implementation

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Abstract - This paper deals with the design and assessment of feasibility and efficiency of a feedback strategy used to reduce the sensitivity of the schedule of hybrid chemical plants with respect to uncertainties and disturbances. The considered plant has two parallel production lines involving both batch and continuous processes with shared and limited resources. The performance and the tuning of the feedback strategy is illustrated with a realistic benchmark Simulator of the plant developed in Matlab/Simulink/Stateflow environment.

Keywords— **on-line scheduling, hybrid system, hybrid automaton, feedback, chemical process**

I. INTRODUCTION

In this paper, we are concerned with the optimal on - line scheduling of hybrid chemical plants in the presence of uncertainties and disturbances, by means of a feedback strategy. The considered benchmark plant has two parallel production lines involving both batch and continuous chemical processes. The particularity of these lines is that they share limited resources (loading of raw material, heating and cooling, intermediate storage, etc). The processes carried out in the plant are hybrid and therefore the plant modeling is done by means of the hybrid automaton formalism [WIL 03], [JOH 03].

The aim of the scheduling is to determine the production plan of the plant, which maximizes its short term productivity. We are solving the problem of the productivity maximization by means of a static cyclic continuous time scheduling. This means that we determine the optimal periodic plan of the plant. This plan defines the optimal production cycle, the starting time of each task, the quantity of resources to use in each of its units, etc. The constraints of the scheduling model are the limited capacity of the shared resources, the coordination of batch and continuous tasks, etc. However owing to the process uncertainties and the disturbances, it is clear that, in practice, the actual processing time of the various modes will be different from that used in the scheduling formulation and therefore the real plant operation according to the initial schedule will be sub - optimal or even infeasible.

The main objective of this paper is to assess the feasibility and efficiency of a feedback strategy in order to reduce the sensitivity of the schedule with respect to disturbances and uncertainties.

The paper is organized as follows: Section 2 presents the hybrid automaton model of the benchmark plant and the statement of the scheduling problem. Section 3 is concerned with the continuous time formulation of the static periodic scheduling of the plant. Section 4 presents the on-line feedback implementation of the periodic plant scheduling in the presence of model uncertainties and process disturbances. The performance and the tuning of the feedback approach is demonstrated with a realistic benchmark Simulator of the plant developed in Matlab/Simulink/Stateflow environment. The results are given in Section 5. Some final comments and directions for future work are presented in Section 6.

II. BENCHMARK PLANT MODEL AND STATEMENT OF THE SCHEDULING PROBLEM

The general structure of the benchmark chemical plant is illustrated in Fig.1. The objective is to maximize the productivity in the presence of uncertainties and disturbances.

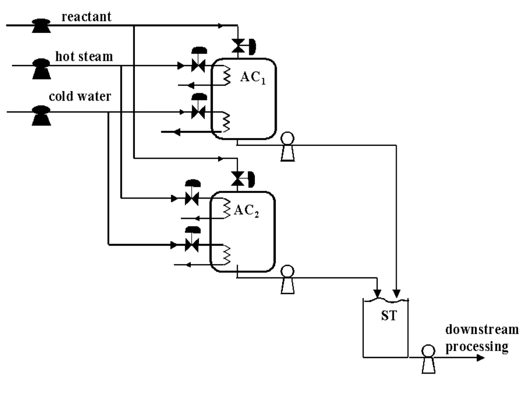

Fig. 1. Benchmark Chemical Plant

Two parallel batch reactors (autoclave one: AC_1 and autoclave two: AC_2) deliver the same final species B produced by a second order exothermic reaction $2.A \longrightarrow B$. The *resources* (reactant, hot steam and cold water) used during the production process are *common and limited*. The final unit of the system is a storage tank (ST) which receives the output product B from each autoclave. The product B is then discharged continuously from the tank to feed in the downstream processing stage. In fact the plant dynamics are more complex than suggested by the simple structure depicted in Fig.1. The plant is actually an hybrid system that combines time driven and event driven dynamics. In this paper, the plant is described by an "Hybrid Automaton Model" which is presented hereafter.

A. Hybrid Automaton Model of Autoclaves

The processes performed in both autoclaves (production lines) are identical and therefore we shall describe only the first one (AC_1). The process follows a sequence of eight successive modes, namely:

TABLE I
MODES IN AC_1

mode name	abbreviation
Stand by (before filling)	StB_{BF_1}
Filling	F_1
Stand by (before heating)	StB_{BH_1}
Heating	H_1
Temperature Regulation	TR_1
Cooling	C_1
Stand by (before discharging)	StB_{BD_1}
Discharging	D_1

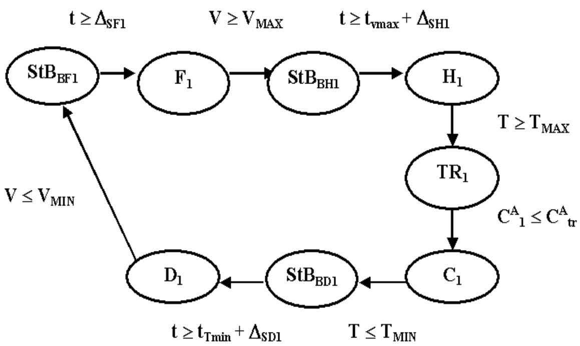

Fig. 2. Hybrid Automaton Diagram of AC_1

The graph of the hybrid automaton for AC_1 is depicted in Fig.2. The vertices of the graph represent the modes (or states) of the automaton while the edges represent the time and/or state event driven transitions between the modes. The process behavior during each mode is characterized by a set of continuous differential equations (mass and energy balances) [LUY 73] with continuous state variables: V_1 $[m^3]$ - volume, T_1 $[K]$ - temperature, C_1^A $[mole/l]$ - concentration of reactant A, C_1^B $[mole/l]$ - concentration of product B. A short description of each mode is made as follows:

• StB_{BF1}*: Stand by before filling*: During this mode there is no resource supply for AC_1, therefore

$$\dot{V}_1 = 0 \quad \dot{C}_1^A = 0 \quad \dot{C}_1^B = 0 \quad \dot{T}_1 = 0 \tag{1}$$

After the time duration Δ_{SF1} $[h]$ defined by the scheduler, AC_1 goes to F_1 mode (Fig.2).

• F_1*: Filling*: During the filling, the raw material *A* is fed in AC_1. This is modeled as follows:

$$\dot{V} = F_1 \quad C_1^A = C_{in}^A \quad C_1^B = 0 \quad T_1 = T_{in} \tag{2}$$

The filling procedure continues until the volume V_1 of AC_1 reaches its maximum value V_{max}, then it starts mode StB_{BH1} (Fig.2).

• StB_{BH1}*: Stand by before heating*: No resource supply for AC_1 and therefore:

$$\dot{V}_1 = 0 \quad \dot{C}_1^A = 0 \quad \dot{C}_1^B = 0 \quad \dot{T}_1 = 0 \tag{3}$$

The time duration of this task Δ_{SH1} $[h]$, is defined by the scheduler. After this time AC_1 goes to H_1 mode (Fig.2).

• H_1*: Heating*: AC_1 is supplied with the hot steam and during this mode the above mentioned second order exothermic reaction $2.A \longrightarrow B$ begins. We model the reaction process as follows:

$$\dot{V}_1 = 0 \quad \dot{C}_1^A = -2k(T_1)(C_1^A)^2 \quad \dot{C}_1^B = k(T_1)(C_1^A)^2$$
$$\dot{T}_1 = -[(\Delta H k(T_1)(C_1^A)^2]/(\rho Cp) + qh_1(T_h - T_1) \tag{4}$$

The heating mode stops when the temperature T_1 of AC_1 reaches a maximum value T_{max}, then it goes directly to the TR_1 mode (Fig.2).

• TR_1*: Temperature Regulation*: Cold water is added into the cooling device of AC_1 in order to remove the heat of the exothermic reaction and to maintain the temperature at T_{max}. Here we use a simplified model for the temperature regulation ($T_1 = T_{max}$):

$$\dot{V}_1 = 0 \quad \dot{C}_1^A = -2k(T_1)(C_1^A)^2$$
$$\dot{C}_1^B = k(T_1)(C_1^A)^2 \quad T_1 = T_{max} \tag{5}$$

The regulation stops when the reactant concentration C_1^A achieves a given threshold value C_{tr}^A, then AC_1 enters immediately in mode C_1 (Fig.2).

• C_1*: Cooling:* Here cold water is added into the cooling device of AC_1 in order to stop the reaction quickly. During this mode we use the following set of differential equations:

$$\dot{V}_1 = 0 \quad \dot{C}_1^A = -2k(T_1)(C_1^A)^2 \quad \dot{C}_1^B = k(T_1)(C_1^A)^2$$
$$\dot{T}_1 = -[(\Delta H k(T_1)(C_1^A)^2]/(\rho Cp) + qc_1(T_c - T_1) \tag{6}$$

The cooling phase continues until the temperature of the autoclave T_1 reaches a minimum value T_{min}. After this task AC_1 enters in StB_{BD1} mode (Fig.2).

• StB_{BD1}*: Stand by before discharging*: No resource supply for AC_1 and therefore:

$$\dot{V}_1 = 0 \quad \dot{C}_1^A = 0 \quad \dot{C}_1^B = 0 \quad \dot{T}_1 = 0 \tag{7}$$

After certain time duration Δ_{SD1} $[h]$ defined by the scheduler, AC_1 goes to D_1 mode (Fig.2).

• D_1*: Discharging*: The discharging of the product B into the storage tank is modeled as follows:

$$\dot{V}_1 = -F_1^{out} \quad \dot{C}_1^A = 0 \quad \dot{C}_1^B = 0 \quad \dot{T}_1 = 0 \tag{8}$$

and it continues until the volume V_1 of AC_1 reaches its minimum value V_{min}. Then AC_1 goes to StB_{BF1} and the production cycle starts again (Fig.2).

Here F_1 $[m^3/h]$ is the volumetric feed flow rate of reactant A, C_{in}^A $[mole/l]$ is the feed flow reactant concentration, T_{in} $[K]$ is the feed temperature, qh_1 $[1/h]$ is the hot steam flow rate, qc_1 $[1/h]$ is the the cold water flow rate, F_1^{out} $[m^3/h]$ is the output volumetric flow rate, $\Delta H < 0$ $[J/mole]$ (exothermic reaction) is the heat released during the reaction, $k(T_1) = k0exp^{-E/(RT_1)}$ $[l/mole.h]$ is the specific reaction rate with *Arrhenius* temperature dependence, E $[J/mole]$ is the activation energy of the reaction, R $[J/mole.K]$ is the gas constant, $k0$ is the reaction rate constant (units same as $k(T_1)$), T_c $[K]$ is the cold water temperature, T_h $[K]$ is the hot steam temperature, ρ $[kg/l]$ is the reactant density , C_p $[J/kg.K]$ is its average specific heat capacity, t [h] is the current time of the process, t_{vmax} [h] is the time when the volume V_1 reaches its maximal value, t_{Tmin} [h] is the time when the temperature T_1 reaches its minimal value.

B. Hybrid Automaton Model of the Storage Tank

The storage tank (ST) is used to transfer continuously the product B to the downstream processing stage. It has five working modes namely:

TABLE II
MODES IN ST

mode name	abbreviation
Stand by	StB
Filling from AC_1 and discharging	$FAC1D$
Filling from AC_2 and discharging	$FAC2D$
Discharging only	D
Production stop	PS

The graph of the hybrid automaton for the storage tank is depicted in Fig.3. As before the vertices of the graph represent the modes of the automaton while the edges represent

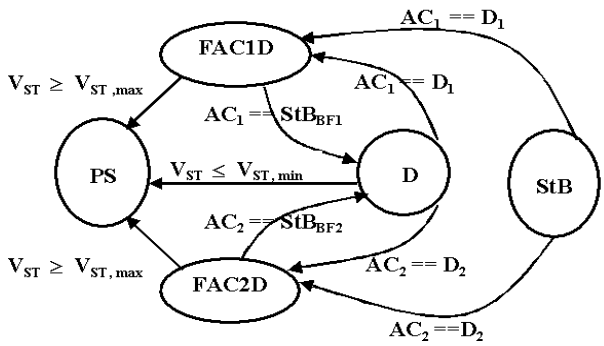

Fig. 3. Hybrid Automaton Diagram of ST

the state and/or external event driven transitions between the modes. The process behavior during each mode is characterized by a set of continuous differential equations (only mass balances) with continuous state variables: V_{ST} $[m^3]$ - volume of the storage tank, C_{ST}^B $[mole/l]$ - concentration of product B in the tank. A short description of each mode is as follows:

• StB*: Stand by mode:* During this mode the tank is empty and its inflow and outflow rates are zero, therefore:

$$\dot{V_{ST}} = 0 \quad \dot{C}_{ST}^B = 0 \tag{9}$$

The StB continues either until AC_1 or AC_2 enters its discharging modes: D_1 and D_2, respectively. Then the tank goes to mode $FAC1D$ or to mode $FAC2D$, respectively (Fig.3).

• $FAC1D$*: Filling from* AC_1 *and discharging*: During this mode the tank is fed with flow rate F_1^{out} and discharged with flow rate F_{ST}^{out}. We model this as follows:

$$\dot{V}_{ST} = F_1^{out} - F_{ST}^{out}$$
$$\dot{C}_{ST}^B = (F_1^{out}/V_{ST}).(C_1^B - C_{ST}^B) \tag{10}$$

This mode continuous either until AC_1 enters in mode StB_{BF1}, then the tank goes to mode D or until the tank volume reaches its maximum value - $V_{ST,max}$ and then the tank enters in PS mode (Fig.3).

• $FAC2D$*: Filling from* AC_2 *and discharging*: During this mode the tank is fed with flow rate F_2^{out} and discharged with flow rate F_{ST}^{out}, which is modeled as follows:

$$\dot{V}_{ST} = F_2^{out} - F_{ST}^{out}$$
$$\dot{C}_{ST}^B = (F_2^{out}/V_{ST}).(C_2^B - C_{ST}^B) \tag{11}$$

This mode continues either until AC_2 enters in mode StB_{BF2}, then the tank goes to mode D or until the tank volume reaches its maximum value - $V_{ST,max}$ and then the tank enters in PS mode (Fig.3).

• D*: Discharging only mode*: During this mode the tank is discharged with flow rate F_{ST}^{out}, that means:

$$\dot{V_{ST}} = -F_{ST}^{out} \quad \dot{C}_{ST}^B = 0 \tag{12}$$

This process continues either until AC_1 or AC_2 enter in mode discharging (D_1 or D_2), then the tank enters in mode $FAC1D$ or mode $FAC2D$, respectively or until the tank volume reaches its minimum value - $V_{ST,min}$ and then the tank enters in PS mode (Fig.3).

• *Production Stop Mode*: During this mode the plant production is completely stopped. We model this as follows:

in the case when the storage thank is completely empty:

$$V_{ST} = 0 \quad C_{ST}^B = 0 \tag{13}$$

or in the case when the storage thank is overflowed:

$$V_{ST} = V_{ST_{max}} \quad C_{ST}^B = C_i^B,\ i \in 1, 2 \tag{14}$$

The overall hybrid automaton chemical plant model can be found in [SIM 05a].

C. Statement of the Scheduling Problem

As was mentioned above the aim of the paper is to assess the feasibility and efficiency of a feedback strategy in order to reduce the sensitivity of the optimal schedule with respect to disturbances and uncertainties. The scheduling problem is solved by means of a static continuous time periodic scheduling model. The overall statement of the considered scheduling problem can be made as follows.

Given the fixed parameters of the plant:

• the nominal processing times of the filling, heating, temperature regulation, cooling and discharging modes;

• the maximal and minimal volume of the autoclaves and the storage tank;

• the amount of reactant, hot steam and cold water available and required for the production of the product B;

• the way of the sharing of the reactant, hot steam and cold water;

• and initial plant states.

The scheduler determines:

• the optimal production cycle;

• the starting times of the modes: filling, heating, temperature regulation, cooling and discharging;

• the stand by times before filling, before heating and before discharging;

• the way the resources are shared;

• the output flow rate of the tank and its volume.

such that the plant productivity is maximized. Maximizing productivity means that the continuous discharging of the product B from the storage tank in order to continuously feed the downstream processing is performed at maximal speed.

The scheduling problem is subject to *constraints* which can be summarized as follows:

• the continuous discharging of the storage tank can not be stopped once started;

• the reactant, hot steam, cold water are limited and shared

• the three successive modes heating, temperature regulation, cooling must be performed successively, without interruption.

III. CONTINUOUS TIME FORMULATION FOR THE STATIC CYCLIC SCHEDULING

The main objective is to obtain a cyclic schedule of the plant maximizing its productivity, i.e. the discharging of the storage tank is performed at maximal rate in order to continuously feed the downstream processing.

Taking into account the process description we define two types of tasks into our scheduling formulation: batch and continuous tasks.

A batch task is a task with a fixed processing time and where the quantity of resources, as well as their utilization rates during execution, are also fixed. So the main decision variable for a batch task is its starting time.

A continuous task is by definition a task that remains active all the time once started - hence it has no processing time - but for which the rate of production is a decision variable.

In our benchmark plant, the batch tasks are the filling, heating, regulation, cooling and discharging of the two autoclaves. Since the two autoclaves are identical, we do not distinguish the tasks performed on these autoclaves.

The single continuous task is the discharge of the storage tank.

In order to solve the optimal cyclic scheduling problem the time horizon is divided in two parts: the first part corresponds to the transient schedule and the second part to the repetitive schedule. This division is interesting when the initial plant state is bad with respect to productivity. The transient schedule is used to escape from this “bad” initial state. We will present only the formulation for the cyclic schedule because the transient schedule formulation can be deduced from it.

In a cyclic scheduling model, the time length of the production cycle is not fixed in advance. Moreover we consider a continuous time formulation based on the state task network representation, to avoid the discretization of the time horizon in a very large number of small time periods.

We base our continuous time formulation on [SCH 99].

We represent in Fig. 4, the time decomposition into events and time slots. An event is defined as the beginning or end of a batch task. Hence, a time slot is the time between two events, i.e. the duration of a mode of the hybrid automaton model. Events and time slots are numbered from 1 up to T. In cyclic scheduling, the event at the end of time slot T is event number 1. The formulation of the optimal scheduling has to take into account the limited availability of the resources: hot steam, cold water, storage tank capacity, number of autoclaves.

This productivity maximization problem is formulated as a sequence of mixed integer linear program (MILP), accord-

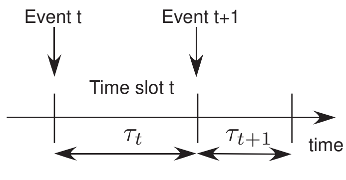

Fig. 4. Event and Time slot

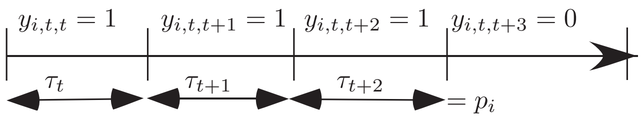

Fig. 5. Processing time

ing to e.g. [ISB 56],[DIN 67] and as it is classical in Data Envelopment Analysis (DEA), see for example [CHA 78]. In the three next subsections, we define the indices and the variables used in our mixed integer linear programming (MILP) formulation, present the constraints, and finally describe the objective function.

A. Indices and variables definition

Here is the definition of the indices and sets that we use in the formulation.

$i \in \{1, \ldots, \text{number of tasks}\}$: index of tasks
$t, t' \in \{1, \ldots, T\}$: indices for events and time slots
BT : set of batch tasks
CT : set of continuous tasks
(singleton in the case of the benchmark plant)

The scheduling variables are the following :

$y_{i,t,t'} (\in \{0,1\})$: $= 1$ if task i started at time slot t and is still active at time slot $t' \geq t$
$= 0$ otherwise [-].
$q_{i,t} (\geq 0)$: is the quantity processed by continuous task i during time slot t [kg].
$\tau_t (\geq 0)$: is the duration of the time slot t [h].

B. The constraints

A detailed mathematical expression of the constraints can be found in [SIM 05b]. In this paper, we limit ourselves to a brief discursive description of the constraints.

1. Timing constraints for the batch tasks
The duration τ_t of time slot t is a variable of the model. The processing time p_i of each batch task i is fixed. Each such task i is executed during a set of consecutive time slots. Therefore a first set of inequality constraints expresses the fact that the processing time p_i must be equal to the sum of the slots durations corresponding to the execution of task i, as shown in Fig. 5.

Another set of constraints states that, if task i is started in time period t and is still active in time period $\Omega(t'+1)$, task i has to be active also in time period $\Omega(t')$:

$$y_{i,t,\Omega(t')} \geq y_{i,t,\Omega(t'+1)} \forall i \in BT, t \in [1,T], t' \in [t, t+T-2] \quad (15)$$

where the periodic function $\Omega(t)$ is defined as follows :

$$\Omega(t) = t \text{ for } t \in [1,T] \quad (16)$$
$$\Omega(t) = \Omega(t-T) \text{ for } t > T \quad (17)$$
$$\Omega(t) = \Omega(t+T) \text{ for } t < 1 \quad (18)$$

A third set of constraints is used to state that, over the cyclic schedule, the different batch tasks have to be performed the same number of times and there is a maximum number of tasks which can be performed simultaneously due to the limited number of autoclaves available. Finally a fourth set of constraints expresses that in some cases it is physically impossible to wait between two tasks i and j.

2. Constraints for the continuous task
The continuous task is the discharge from the storage tank and is active during the whole time horizon of the cyclic schedule.

The following constraints impose that the quantity processed in each no zero duration time period has to fall between some limits:

$$\underline{\rho}_i \tau_t \leq q_{i,t} \leq \bar{\rho}_i \tau_t \forall i \in CT, t \in [1,T] \quad (19)$$

where $\underline{\rho}_i$ and $\bar{\rho}_i$ represent the minimum and maximum production rate of the continuous task i [kg/h].

3. Resource constraints
For each resource, we model the resource availability constraints. For hot stream and cold water, the instantaneous utilization rate is fixed by the scheduling decisions and is constrained by some upper limit (non renewable resource). For the storage tank, the stock level at the start and end of each time slot has to fall between zero and the capacity.
For the autoclaves, we impose the precedence relations between the different tasks. For instance, heating can only and has to be performed immediately after filling.
Modeling these restrictions requires the addition of new variables and constraints.

C. The objective

The objective is to maximize the relative cycle production, or productivity. The nonlinear objective function expressing this idea is the following :

$$\max \frac{\sum_{i \in CT} \sum_{t=1}^{T} q_{i,t}}{\sum_{t=1}^{T} \tau_t} \quad (20)$$

This objective function can be linearized by using the following linear expression :

$$z(\mu_p) = \max \sum_{i\in CT} \sum_{t=1}^{T} q_{i,t} - \mu_p \left(\sum_{t=1}^{T} \tau_t \right) \qquad (21)$$

It is known, see [ISB 56],[DIN 67], that if we solve a series of linear problems with the objective function $z(\mu_p)$ and optimal solution $(q^\star, \tau^\star)$ and if we update μ_p by

$$\mu_{p+1} = \frac{\sum_{i,t} q^\star_{i,t}}{\sum_t \tau^\star_t} \qquad (22)$$

at each step, then the sequence of solutions converge to the optimal solution of the problem with the non linear objective (20).

IV. FEEDBACK PERIODIC SCHEDULING IN THE PRESENCE OF UNCERTAINTIES AND DISTURBANCES

The aim of this communication is to assess the feasibility and the efficiency of a feedback strategy in order to reduce the sensitivity of the optimal periodic schedule with respect to model uncertainties and process disturbances in the chemical plant. The general structure of the feedback control system that we consider is depicted in Fig.6. The inner loop represents the classical feedback control of the reactor temperature by means of the cold water flow rates q_{c1} and q_{c2}. The outer loop is a feedback control loop for on - line scheduling inspired by the classical MBPC (model based predictive control) approach. The optimal scheduling is computed over a rather large prediction horizon but it is implemented with a receding horizon strategy. This means that the new optimal schedule is recomputed regularly on the basis of the available feedback measurements of the actual plant state.

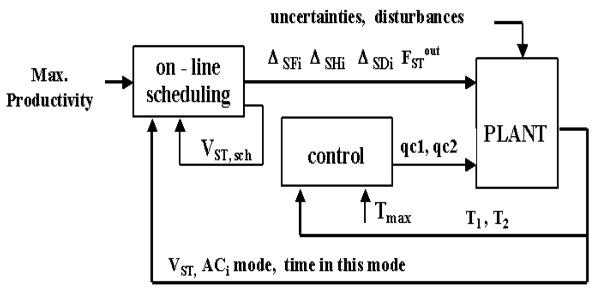

Fig. 6. Feedback Scheduling Structure

The feedback rescheduling strategy that we apply to the plant is described as follows. The initial optimal periodic schedule is computed and applied to the autoclaves (AC_1 and AC_2) by means of the stand by times Δ_{SFi}, Δ_{SHi} , Δ_{SDi}, $i \in 1, 2$ and to the storage tank by its output flow rate F^{out}_{ST}. At the beginning and the end of each time slot (τ_t) the actual volume of the storage tank V_{ST} is compared with the theoretical volume V^{SCH}_{ST} received from the scheduler. When the difference between them becomes larger than a given threshold value, a new optimal schedule is recomputed and applied to the plant. In order to update the optimal periodic schedule, the scheduler uses the following feedback measurements of the current plant state: the volume in the storage tank V_{ST}, the current mode of the production process in each autoclave AC_i, $i \in 1, 2$ and the time elapsed since the beginning of this mode.

V. SIMULATION RESULTS

In order to illustrate the feedback on-line scheduling strategy, a simulator of the considered hybrid chemical plant has been developed in a Matlab/Simulink/Stateflow environment (see [SIM 05b]). It has been simulated under the following conditions:

$F_1 = 162$ $[m^3/h]$, $F^{out}_1 = 162$ $[m^3/h]$, $qh_1 = 3$ $[1/h]$, $T_h = 380$ $[K]$, $qc_1 = 2$ $[1/h]$, $T_c = 280$ $[K]$, $\Delta H = -90000$ $[J/mole]$, $E = 40200$ $[J/mole]$, $R = 8.314[J/mole.K]$, $k0 = 10000$ $[l/mole.h]$, $\rho = 0.9$ $[kg/l]$, $C_p = 1000$ $[J/kg.K]$, $C^A_{in} = 10$ $[mole/l]$, $T_{in} = 300$ $[K]$, $V_{max} = 27$ $[m^3]$, $V_{min} = 0$ $[m^3]$, $T_{max} = 400$ $[K]$, $C^A_{min} = 2$ $[mole/l]$, $T_{min} = 0$ $[K]$, $V_{ST,max} = 50$ $[m^3]$, $V_{ST,min} = 0$ $[m^3]$

Three case studies are successively considered.

- *Exact model of the plant and no process disturbances*

The optimal schedule is computed under the additional assumptions that the filling of the storage tank is instantaneous. The model equations (10), (11) are modified accordingly. The optimal schedule is then applied to the simulator and the simulation results are presented in Fig. 8 and Fig. 9. The values of the corresponding optimal stand by times Δ_{SFi}, Δ_{SHi} , Δ_{SDi}, $i \in 1, 2$ received from the scheduler can be deduced from Fig. 8 and the scheduled values of the output flow rate of ST - F^{out}_{ST} are given in Fig.7. As

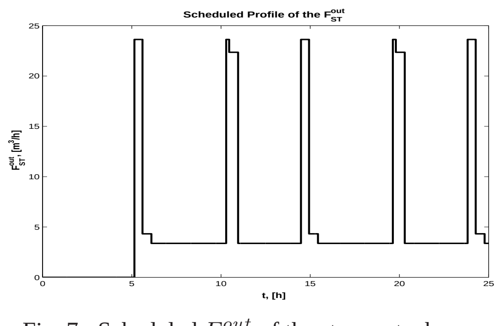

Fig. 7. Scheduled F^{out}_{ST} of the storage tank

expected there is no difference between the real and scheduled starting times in both autoclaves (See Fig. 8). There are three batches produced in AC_1 and two in AC_2. From Fig. 9 it is observed that the real and scheduled volumes in the storage tank coincide. As a result the optimal periodic schedule is achieved and therefore the plant production is maximized over the considered production horizon of $25h$.

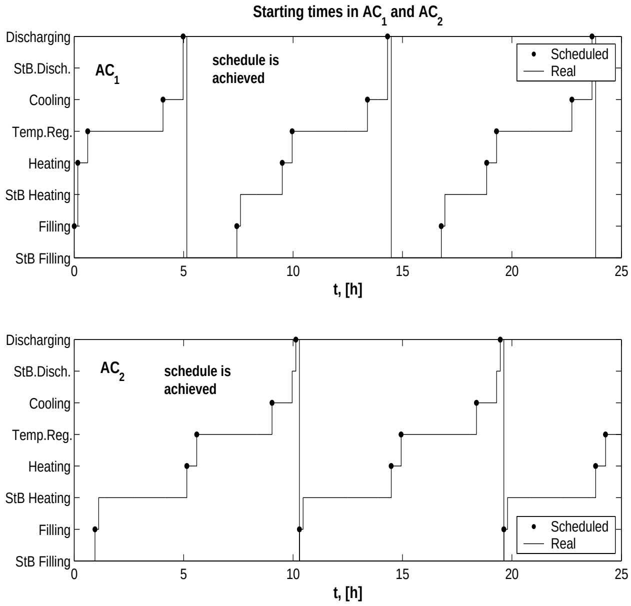

Fig. 8. Real and scheduled starting times in AC_1 and AC_2 with exact model and no process disturbances

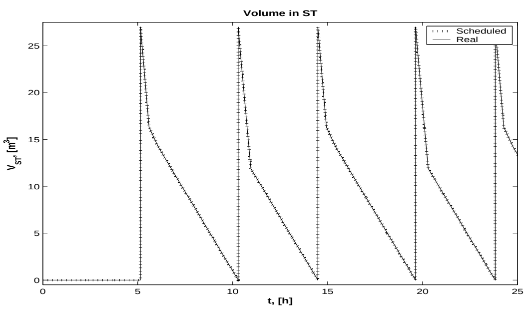

Fig. 9. Real and scheduled volume in ST with exact model and no process disturbances

- *Model uncertainties and process disturbances*

The simulation results for the second case study are depicted in Fig. 10 and Fig. 11. Here it is assumed that the storage tank is fed with the dynamics as modeled by equations (10)-(11) and not instantly as defined in the scheduling formulation. From Fig. 11 it is seen that at $t \approx 14h$ the volume of tank becomes larger than expected. We also assume that at $t = 4h$ the temperature of the hot steam Th is incidentally decreased by $15\ K$. This implies that the duration of the heating mode of each autoclave increases in time with respect to the schedule and therefore that the real starting times of the modes of the chemical production process in both autoclaves are delayed and occur later than scheduled (see Fig. 10). As a result there is also a delay in the feeding of the tank and therefore the tank volume decreases in time. The presence of these modelling uncertainties and process disturbances induce that the optimal schedule is not achieved (Fig. 10) and therefore that the real plant operation according to the initial schedule becomes suboptimal or, in this case, even not feasible. Indeed, in this example, due to the delay of the product B formation at $t \approx 20h$ (see Fig. 11), the tank is not longer fed with product B and the production is stopped. There are only two batches produced in AC_1 and one in AC_2 before this breakdown of the plant.

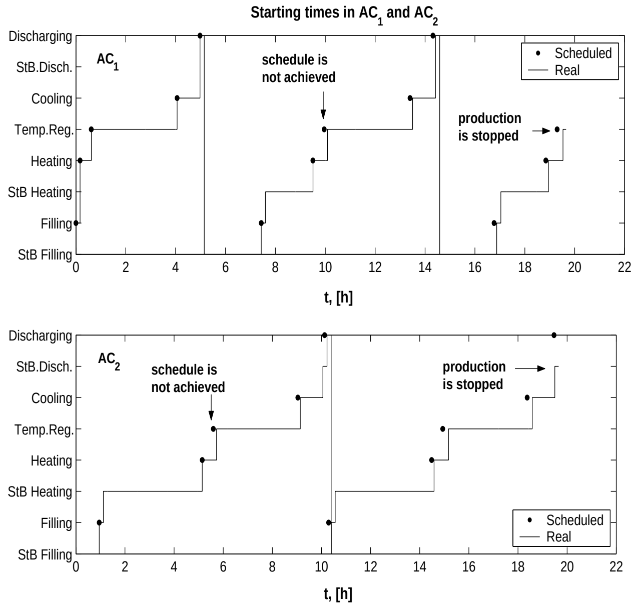

Fig. 10. Real and scheduled starting times in AC_1 and AC_2 with model uncertainty and process disturbance

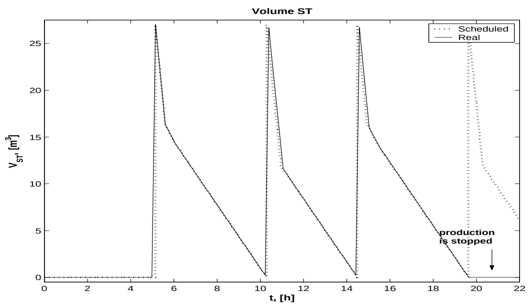

Fig. 11. Real and scheduled volume in ST with model uncertainty and process disturbance

- *Feedback strategy for on-line rescheduling in the presence of model uncertainties and process disturbances*

In order to anticipate and to overcome such an accidental production breakdown, we test the developed on-line feedback rescheduling strategy. As seen from Fig. 11 at $t \approx 10$ h, the process supervisor may easily detect a small difference between the scheduled and real volume profile. A signal is then given to the scheduler to make a rescheduling,

starting at the current measured process state. The feedback measurements received by the scheduler are that AC_1 is in TR_1 mode for a time interval $\Delta_{TR_1} = 0.043\ h$ and AC_2 is in StB_{BD2} mode for $\Delta_{StB_{BD1}} = 0.079\ h$ while the current volume of ST is $0.55 m^3$ (See Fig. 10 and Fig. 11). The new schedule is then applied to the plant as illustrated in Fig. 12. It can be observed that the time durations of the stand by modes in the autoclaves: Δ_{SFi}, Δ_{SHi} and Δ_{SDi}, $i \in 1, 2$ are reduced with the new schedule and therefore that the deviation between the real and the scheduled operations is reduced. But obviously, due to the model uncertainties and process disturbance, this deviation persists and, as a consequence, the difference between the real and the scheduled volume of the storage tank is also progressively increasing (See Fig. 13). But the behaviour of the plant has clearly been improved, since the breakdown occurs now much later at $t \approx 35h$. In addition, comparing with Fig. 10 and 12, it is seen that there are two more batches produced in each autoclave before the plant breakdown.
This simulation result clearly demonstrates the feasibility and potential efficiency of the developed feedback strategy for on-line scheduling.

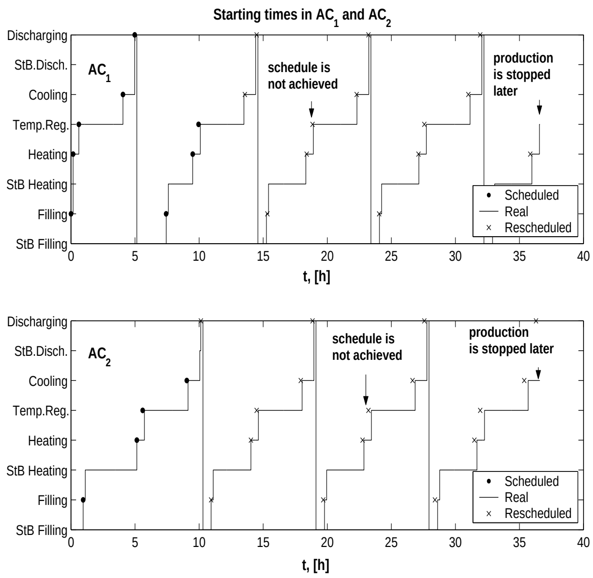

Fig. 12. Real and scheduled starting times in AC_1 and AC_2 with model uncertainty and process disturbance after rescheduling

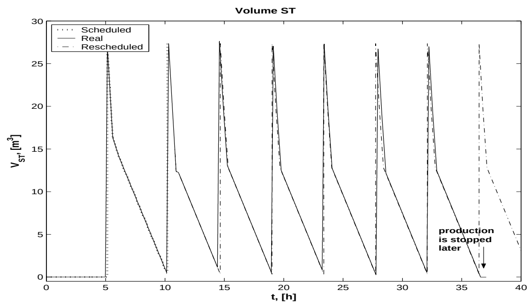

Fig. 13. Real and scheduled volume in ST with model uncertainty and process disturbance after rescheduling

VI. CONCLUSIONS AND FUTURE WORK

An hybrid automaton model for a chemical plant with two parallel production lines and shared resources has been developed and implemented in a Matlab/Simulink/Stateflow environment. A periodic continuous time scheduling for maximizing the plant productivity is available. In this paper, the simulation results have shown that a feedback strategy for on-line rescheduling of the plant is feasible and seems to be potentially efficient in order to maintain a productivity close to optimality in presence of modelling uncertainties and process disturbances. Although the results are promising, there is still a lot of open issues that we intend to investigate. In order to have a flexible feedback control tool for the plant, it is important to get good scheduling solutions from the scheduler quickly enough. In order to achieve such an objective for large and real life plants, it is important to rely on a strong or tight formulation. Our continuous time formulation is currently very weak in the sense that its linear relaxation is far from the convex hull of mixed integer solutions. However, we are still currently improving this formulation in order to design faster scheduling algorithms. Another issue that we will investigate is to be able to give a convergence analysis of the on-line feedback scheduling method, at least in case of simple plants.

Acknowledgments: Work partially done in the frameworks of the "Fonds de Recherche SOLVAY", "HYCON Network of Excellence,contract number FP6-IST-511368" and "Belgian Programme on Interuniversity Attraction Poles, initiated by the Belgian Federal Science Policy Office"

REFERENCES

[CHA 78] CHARNES A., COOPER W., RHODES E., *Measuring the efficiency of decision making units*, p. 429-44, European Journal of Operations Research, 1978.

[DIN 67] DINKELBACH W., *On nonlinear fractional programming*, vol. 13, p. 492–498, Management Science, 1967.

[ISB 56] ISBELL J., MARLOW W., *Attrition Games*, vol. 3, p. 71–93, Naval Research Logistics Quarterly, 1956.

[JOH 03] JOHANSSON K., *EECS291E Hybrid Systems, http://www.s3.kth.se/ kallej/eecs291e/*, 2003.

[LUY 73] LUYBEN W., *Process Modeling, Simulation, and Control for Chemical Engineers*, McGraw-Hill, Inc., 1973.

[SCH 99] SCHILLING G., PANTELIDES C., *Optimal periodic scheduling of multipurpose plants*, vol. 23, p. 635–655, Computers Chemical Eng., 1999.

[SIM 05a] SIMEONOVA I., WARICHET F., BASTIN G., DOCHAIN D., POCHET Y., *Benchmark Hybrid Automaton Model of Industrial Process*, http://astwww.bci.uni-dortmund.de/hycon4b/, 2005.

[SIM 05b] SIMEONOVA I., WARICHET F., BASTIN G., DOCHAIN D., POCHET Y., *On - line Scheduling of Hybrid Chemical Plants with Parallel Production Lines and Shared Resources: a Feedback Control Implementation*, Extended version of this paper, CESAME/CORE Internal report, 2005.

[WIL 03] WILLEMS J., *Lecture Notes in Hybrid Systems*, UCL - Belgium, Graduate School in Systems and Control, Spring, 2003.